IFLST 3 Unions and intersections of families of sets

3.1 Find $\bigcap_{i\in\mathbb{N}} A_i$ for:

a) $A_i = (0; \frac{1}{i+1})$, b) $A_i = (0; \frac{1}{i+1}]$, c) $A_i = [0; \frac{1}{i+1}]$, d) $A_i = (i; \infty)$

3.2 Find $\bigcup_{i\in\mathbb{N}} A_i$ for:

a) $A_i = (i; \infty)$, b) $A_i = (\frac{1}{i+1}; 1 - \frac{1}{i+1}]$; , c) $A_i = (-i; i)$

3.3.Find:

a) $\bigcap_{i\in\mathbb{N}_+}[2 - \frac{1}{(i+3)^2+2}, 7 + \frac{1}{(i-2)^2+2}) =$ b) $\bigcup_{i\in\mathbb{N}_+}[2 - \frac{1}{(i+3)^2+2}, 7 + \frac{1}{(i-2)^2+2}) =$

c) $\bigcap_{i\in\mathbb{N}_+}[1 + \frac{1}{(i-3)^2+2}, 5 - \frac{1}{(i+2)^2+2}) =$ d)$\bigcup_{i\in\mathbb{N}_+}[1 + \frac{1}{(i-3)^2+2}, 5 - \frac{1}{(i+2)^2+2}) =$

e) $\bigcap_{i\in\mathbb{N}_+}[1 + \frac{1}{2i}, 5 - \frac{1}{i}) \times [1 - \frac{1}{2i}, 5 - \frac{1}{i}] =$

f) $\left(\bigcup_{i\in\mathbb{N}_+}[1 + \frac{1}{2i}, 5 - \frac{1}{i})\right) \times \left(\bigcup_{i\in\mathbb{N}_+}[1 - \frac{1}{2i}, 5 - \frac{1}{i}]\right) =$

g)$\bigcap_{i\in\mathbb{N}_+}[1 - \frac{1}{3i}, 5 - \frac{1}{i}) \times [1 + \frac{1}{3i}, 5 - \frac{1}{i}] =$

h)$\left(\bigcup_{i\in\mathbb{N}_+}[1 - \frac{1}{3i}, 5 - \frac{1}{i})\right) \times \left(\bigcup_{i\in\mathbb{N}_+}[1 + \frac{1}{3i}, 5 - \frac{1}{i}]\right) =$

3.4 Find $\bigcap_{i\in\mathbb{N}} A_i$ and $\bigcup_{i\in\mathbb{N}} A_i$ for:

a) $A_i = \{(x, y) \in \mathbb{R}^2 : x + y \leqslant i\}$,

b) $A_i = \{(x, y) \in \mathbb{R}^2 : x + y \leqslant \frac{1}{i+1}\}$,

c) $A_i = \{(x, y) \in \mathbb{R}^2 : x, y \geqslant 0, y \leqslant x^i\}$.

3.5 Find: a) $\bigcup_{a\in(0,\infty)}[a, \infty) \times (-\infty, a]$ b) $\bigcap_{a\in[0,1]}[a, \infty) \times (-\infty, a]$

3.6 Find $\bigcup_a \bigcap_b X_{a,b}$, $\bigcap_a \bigcup_b X_{a,b}$, $\bigcap_b \bigcup_a X_{a,b}$, $\bigcup_b \bigcap_a X_{a,b}$ for:

a) $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : 0 < x \ \wedge 0 \leqslant y \ \wedge \frac{x}{a} + \frac{y}{b} \leqslant 1\}$, $a, b \in \mathbb{R}$, $a, b > 0$,

b) $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : y \leqslant a(x - b)\}$, $a, b \in \mathbb{R}$,

c) $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : y \leqslant ax(x - b)\}$, $a, b \in \mathbb{R}$, $a, b > 0$,

d) $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : y \geqslant \frac{a}{b^2}x(x - 2b)\}$, $a, b \in \mathbb{R}$, $a, b > 0$,

e) $X_{a,b} = \{x \in \mathbb{R} : a - \frac{1}{b} \leqslant x < a + \frac{1}{b}\}$, $a \in \mathbb{Z}$, $b \in \mathbb{N} - \{0\}$),

f) $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : 0 < y \leqslant ax^2 + b\}$, $a, b \in \mathbb{R}$, $a < 0$, $b > 0$,

g) $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : y = a(x - b)^3 + b\}$, $a, b \in \mathbb{R}$, $a, b > 0$,

h) $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : y \geqslant e^{a(x-b)}\}$, $a, b \in \mathbb{R}_+$, $a, b > 0$,

i*) $X_{a,b} = \{(x, y) \in \mathbb{R}^{+2} : ax^2 < y \leqslant \sqrt[3]{ab^2}\}$, $a, b \in \mathbb{R}$, $a, b > 0$.

3.6 Prove:

a) $(\bigcup_{i\in I} A_i) \times (\bigcup_{j\in J} B_j) = \bigcup_{i\in I} \bigcup_{j\in J}(A_i \times B_j)$,

b) $(\bigcap_{i\in I} A_i) \times (\bigcap_{j\in J} B_j) = \bigcap_{i\in I} \bigcap_{j\in J}(A_i \times B_j)$,

3.7 Find infinite family $\mathcal{X}$ of subsets of $\mathbb{N}$ such, that $\bigcap \mathcal{X} = \emptyset$ and $\bigcap \mathcal{Y} \neq \emptyset$ for every proper subfamily $\mathcal{Y}$ of the family $\mathcal{X}$.